UNIVERSITY OF NORTH BENGAL

समानो मन्त्रः समितिः समानी

Syllabus for

PhD Course Work

in

MATHEMATICS

Department of Mathematics, University of North Bengal, Raja Rammohunpur, P.O. - N.B.U.,
Dist. - Darjeeling, West Bengal, India, Pin-734013

S. Ghosal
15/01/2021.
Signature of HOD
Department of Mathematics, NBU

HEAD
Department of Mathematics
University of North Bengal

Structure of Syllabus for PhD Course Work in Mathematics

Course Code	Course Type	Course Name	Full Marks (External)	Full Marks (Practical)	Full Marks (Internal)	Full Marks (Total)	Credit
PHDMATH-101	**Theory**	**Research Methodology: Research Foundation**	**40**		**10**	**50**	**2**
PHDMATH-102	**Practical**	**Research Methodology: Computer Application in Research**		**40**	**10**	**50**	**2**
PHDMATH-103	**Theory**	**Elective:** Any two papers to be chosen from **Table-I**, based on research interest of the students and availability of suitable teachers/ Supervisors.	**40 + 40**		**10 + 10**	**50 + 50**	**2 + 2**
		Total	**160**		**40**	**200**	**8**

Any two papers to be chosen from the list of electives (Table: I) based on the research interest of the scholar and availability of suitable teacher/supervisor

- **One Credit shall mean one teaching period of 1 (one) hour per week for one semester (of 15 weeks) for theory courses and 2 (two) practical/laboratory/field/demonstration hours/week for one semester.**
- **Pass marks will be 55% or equivalent grade in each paper.**

Table: I

Elective Papers for PHDMATH-103 (M1) to PHDMATH-103 (M18)	
Elective Paper Sub-Code	**Title of the Paper**
M1	**Theory of Convergence and Topological Hyperalgebra**
M2	**Basic Theory of Entire and Meromorphic Functions**
M3	**Measure Theory**
M4	**Summability Methods**
M5	**Topics in Analytic Number Theory**
M6	**Topological Indices of Graphs**
M7	**Advanced Modern Algebra**
M8	**Finite and Boundary Element Methods**
M9	**Similarity Transformations and Perturbation Theory**
M10	**Advanced Numerical Techniques and MATLAB**
M11	**Modern Theory of Partial Differential Equations**
M12	**Advanced Trends in Continuum Mechanics**
M13	**Advanced Fluid Mechanics**
M14	**Mathematical Methods**
M15	**Optimization Techniques**
M16	**Decision Theory**
M17	**Differential Geometry of Manifolds**
M18	**Mathematical Theory of Elasticity**

<u>Detailed Syllabus</u>

Ph.D. Course Work

PHDMATH-101: Research Methodology: Research Foundation **Full Marks: 25 Credit: 2**

Introduction- Meaning, purpose, objectives, characteristics, motivation, significance, types of research; approaches, process, methods and methodology used in research ; criteria of good research; research methods in general and Mathematical Sciences in particular.

The Research Problem – Research problems and sub-problems identification, stating, defining, techniques involved in defining problem.

Interpretation and Report Writing- Techniques of interpretation- significance, types, steps, checklist/precautions and characteristics of research documentation i.e. reviews, treatise, monographs, abstracts, articles, technical reports, white papers, research papers, thesis etc.- issues and techniques of writing project proposals, paper presentation and soft skills.

Ethical issues and Professional Conduct- Ethics in general, Professional Ethics, ethical Issues and their significance those arise from Computer Technology, General Moral Imperatives, Concepts and issues related to plagiarism and Intellectual Property Rights

Literature Review – Importance of Literature review in defining a problem, including literature in research proposal, critique, survey & peer review process, identifying gap areas from literature review; Major Research areas, Journals, Publication, Conferences and Status of Research in the field of Mathematical Sciences.

Extensive review work has to be undertaken by the candidate in relevance to his/her topic of research interest. The candidate will be required to submit a written report on the survey to be evaluated by the Departmental Council. Candidate will also be required to deliver a seminar lecture on the survey work.

<u>References:</u>

1. *Research Methodology Methods Techniques by C. R. Kothari, Wishwa Prakashan Publishers.*
2. *An introduction to Research Methodology by Garg, B.L., Karadia, R., Agarwal, f. and Agarwal, U.K., RBSA Publishers.*
3. *Research Methodology Sinha, S.C. and Dhiman, A.K.,Ess Publications. 2 volumes.*
4. *Research Methods: the concise knowledge base by Trochim, W.M.K., Atomic Dog Publishing.*
5. *How to Write and Publish a Scientific Paper by Dey, R.A., Cambridge University Press.*
6. *Conducting Research Literature Reviews: From the Internet to Paper by Fink, A., Sage Publications*
7. *Proposal Writing by Coley, S.M. and Scheinberg, C.A., Sage Publications, 1990.*
8. *Handbook on Intellectual Property Law and Practice by Subbarau NR, S Viswanathan, Printers and Publishing Private Limited*
9. *Research Methodology by Dalip Kumar Bhattacharya*
10. *Research Methodology by C.H Chaudhary, RBSA Publication*
11. *Statistical Techniques by S.P. Gupta, Sultan Chand & Sons*
12. *An Introduction to Multivariate Statistical Analysis by Andreson T. W., Wiley Eastern Pvt., Ltd., New Delhi.*
13. *WWW(Web Sources)*

PHDMATH-102: Research Methodology: Computer Application in Research

Full Marks: 25 Credit: 2

Historical background of LATEX and its pronunciation:
Creating a new document, Opening and saving a Doument, Use of document classes like article.cls, amsart.cls, books.cls, report.cls; Adding pckages like amssymb, amsmath, amsthm, amsfonts, graphics, graphix, times;

Entering Text, Editing Text

Entering and Editing Mathematics: Entering Mathematical Characters, Entering Mathematical Objects, Entering Mathematics with Fragments, Using Body Math, Editing Mathematics

Formatting Your Document: Formatting with tags: theorem, definition, corollary, lemma, proposition, example, acknowledgement, axiom, proposition and changing their styles; Formatting the Page, Changing body text point size, paper size, orientation, title page, two column category, equation number position; Use of unit of measurement: ex, em, pt, cm, mm, in; Definition of commands : \newcommand, \renewcommand, \setcounter, \addcounter, \newenvironment, \renewenvironment; Introduction to spacing: horizontal spacing, vertical spacing, small space, big space etc; Different pagebreaks: newline, line break; Making section, subsection etc, Alignment of texts, pictures, creating post script files (*.ps, *.eps) and inserting them into Tex files.

Saving Your Documents: **Saving Portable LATEX Files, Exporting Files.**

Previewing and Printing Your Document: Creating PDF Files, Exporting Documents as HTML Files.

Special features of Scientific Workplace: Computing and Plotting, Evaluate and Evaluate Numerically, Factor, Combine, Expand, Simplify, Equality, Solve Exact and Numeric, Work with Polynomials, 2-D and 3-D Plots, Compute in Place, Perform Matrix Operations, Solve Differential Equations, Compute Statistics, Compute with Units of Measure, Create Exams and Quizzes.

Typesetting Your Document: Understanding the Typesetting Process, Typeset Previewing and Typeset Printing, Understanding the Appearance of Typeset Documents, Creating Typeset Document Elements, Creating Cross-References, Creating Notes, Creating Bibliographies and Citations, Obtaining More Information about Typesetting.

Basics of operating systems: handling different operating systems, Literature survey using web, handling search engines.

Computer usage for collecting/analyzing data: simulations using fortran/ C/ Mathematica/ MATLAB/Mathcad----Solving Differential equation, numerical solution of differential equation, Integration, Solution of Algebraic equation, numerical solution of differential equation, Integration, Solution of Algebraic equation; Plotting of solution curve of system of differential equations, Two- and three-dimensional plotting, plotting data, using options, and creating dynamic and interactive graphics.

Preparing presentations:

i) Research papers: Using word processing software – MS Word/Latex/others, Drawing graphs and diagrams – Origin/Xmgrace/Excel/others.

ii) Seminar presentations – Beamer/Power point for oral and poster presentations.

Use of Acrobat reader, Texnic Centre, WinEdt, Ghost View; Conversion of graphics in different formats by Ghost View.

References:

1. *A document preparation system- Leslie Lamport, Addison Weseley Publisher Company, 1994*
2. *Getting Started with Scientific Workplace®, Scientific Word® & Scientific Notebook® Version 5- Susan Bagby, Publisher: MacKichan Software*
3. *The Computer Science of TeX and LaTeX by Victor Eijkhout*

4. *LATEX for Biginners: Workbook [Available at http://www.docs.is.ed.ac.uk/skills/documents/3722/3722-2014.pdf]*
5. *LATEX Tutorials A PRIMER Indian TEX Users Group Trivandrum, India 2003 September Edited by E. Krishnan [Available at https://www.tug.org/twg/mactex/tutorials/ltxprimer-1.0.pd]*
6. *LaTeX Wikibooks.org/wiki/LaTeX]*
7. *LaTeX Beginner's Guide by Stefan Kottwitz*
8. *Getting Started with LaTeX by David R. Wilkins*

M1: Theory of Convergence and Topological Hyperalgebra **Full Marks: 50 Credit: 2**

Theory of Convergence

A Generalized Statistical Convergence via Ideals, I and I*convergence in topological spaces, A note on I-convergence and I*-convergence of sequences and nets in topological space, I and I* convergence of nets, On I-convergence of nets in locally solid Riesz space, Some further results on I-Cauchy sequences and condition (AP), I-Lambda statistical convergence in topological groups, When I-Cauchy nets in complete uniform spaces are I-convergent, Some Further results on Ideal Convergences in Topological Spaces, I-Lambda Statistically convergent sequences in topological groups, On I-Cauchy nets and Completeness, When I-Cauchy nets in complete uniform spaces are I-convergent, I^K- convergence, I^K-Cauchy Functions, Extending Asymmetric convergence and Cauchy conditions using ideals.

Topological Hyperalgebra:

Introduction to Topological Groups and Semigroups:

Semitopological groups: The concept of a semitopological group, neighborhood system of identity, constructions of new semitopological groups from old, locally compact semitopological groups.
Topological group: Translation in topological groups, neighborhood system of identity, separation axioms in topological groups, uniform structure on a topological group, sub groups, quotient groups, locally compact groups, continuous and open homomorphism, open homomorphism and closed graph theorems.

Introduction to Topological Rings:

Definition, examples of topological rings, topological modules, vector space, and algebras, neighborhoods of zero, subrings, ideals, submodules.

Metrizable groups, completions of commutative Hausddorff groups, completions of topological ring and modules, locally bounded modules and rings.

Introduction to algebraic hyperstructures:

What algebraic hyperstructures are? A historical development of algebraic hyperstructures.
The hypergroup of Marty: Definitions, examples, subhypergroups, Some kinds of subhypergroups, homomorphism of hypergroups, Join spaces, Canonical hypergroups, Polygroups, regular and strongly regular relations, the fundamental relation, complete parts, the heart of a hypergroup, complete semihypergroups, complete hypergroups, Polygroups: Difinition, examples, extension of polygroups by polygroups, subpolygroups and quotient polygroups , isomorphism theorems of polygroups, generalized permutation, permutation polygroups, representation of polygroups, solvable polygroups, nilpotent polygroups.Thehyperring of Krasner: Definition and constructions of Krasner hyperrings, hyperideals, quotient hyperrings and homomorphisms, special hypeideals.

Introduction to topological algebraic hyperstructures:

Topological Hypergroups in the sense of Marty: Definitions, examples, topological subhypergroup, compact Hausdorff topological hypergroup.

Topological Polygroups; Definition, examples, results, subpolygroups of a topological polygroup, isomorphism theorems, rolls of complete parts in topological polygroups, left big subsets of topological polygroups.

Topological Polygroups: Definition, examples, related identity, some results.

References:

1. *T. Husain, Introduction to Topological Groups, W. B. Saunders Company, 1966.*
2. *A. Arhangel'skii, M. Tkachenko, Topological Groups and Related Structures, Atlantis Press/Word Scientific, 2008.*
3. *G. McCarty, Topology: An Introduction with Application to Topological Groups, Dover, 2011.*
4. *B. Davvaz, V. Leoreanu-Fotea, Hyperring Theory and Applications, International Academic Press, 115, Palm Harber, USA, 2007.*
5. *B. Davvaz, Polygroup theory and Related Systems, Word Scientic Publishing Co. Pte. Ltd., Hackensack, NJ, 2013.*
6. *D. heidari, B. Davvaz, S. M. S. Modarres, Topological hypergroups in the sense of Marty, Communications in Algebra, 42 (2014) 4712-4721.*
7. *D. Heidari, B. Davvaz, S. M. S. Modarres, Topological polygroups, Bull. Malays. Math. Sci. Soc., 39 (2016) 707-721.*
8. *M.S Shadkami, M.R. AhmadiZand, B. Davvaz, The Role of Complete Parts in Topological Pologroups, Int. J. Anal. Appl., 11(1)(2016) 54-60.*
9. *M. SalehiShadkami, M. R. AhmadiZand and B. Davvaz, Left big subsets of topological polygroups, Filomat, 30:12 (2016), 3139-3147.*
10. *M. Singha, K. Das, B. Davvaz, On Topological Complete Hypergroups, Filomat 31 (2017), no. 16, 5045–5056.*

M2: Basic Theory of Entire and Meromorphic Functions **Full Marks: 50 Credit: 2**

Elementary theory of entire and meromorphic functions, Poission –Jensen formula, Nevanlinna characteristic function, Nevanlinna's first fundamental theorem, Cartan's identity, Orders of growth, Comparative growth of T(r) and log M(r), Weierstrass product, Representation of a meromorphic function in terms of its zeros and poles, Nevanlinna's second fundamental theorem, Various types of deficiencies of meromorphic functions and their properties.

References:

1. *W. K. Hayman: Meromorphic functions, The claredon Press, Oxford, 1964.*
2. *C. C. Yang and H. X. Yi: Uniqueness Theory of Meromorphic Functions, Science Press, Beijing, 2003.*
3. *G. Valiron: Lectures on the general theory of integral functions, Chelsea Publishing Company, 1949.*

M3: Measure Theory **Full Marks: 50 Credit: 2**

Lebesgue Measure space on the Euclidean space. Lebesgue Outer Measure on the Euclidean space. Definition and properties. Regularity properties; Borel measure space of n – dimension; Completion of Borel Measure space; Estimation of Lebesgue measure on $\mathbb{R}^n$byclosed and compact sets and approximation by open sets.

Hausdorff Measures, Regularity and Hausdorff dimension, Hausdorff measures of integral and functional dimensions.

References:

1. *Lectures on Real Analysis by J. Yeh, World Scientific.*
2. *Theories of Integration by Douglas S. Kurtz & Charles W. Swartz, World Scientific.*

M4: Summability Methods **Full Marks: 50 Credit: 2**

Density, asymptotic density/ natural density, statistical convergence, statistical boundedness, statistical limit point, statistical cluster point, relation between statistical limit points set and statistical cluster points set, statistical limit superior and limit inferior, lacunary statistical convergence, strongly Cesaro summability, natural density of order α (where $0 < \alpha \leq 1$), statistical convergence of order α, λ-statistical convergence, strong (V, λ)-summability, $\alpha\beta$-statistical convergence, density by moduli, f-statistical convergence, f-statistical limit point, f-statistical cluster point, relation between f-statistical limit points set and f-statistical cluster points set, weighted statistical convergence, statistical convergence in probability.

Double natural density, statistical convergence of double sequences, statistical Cauchy sequence, statistical convergence of multiple sequences, relation between statistical convergence and strongly Cesaro summable sequences, statistical limit points of double sequences, double density of order (α, β) (where $0 < \alpha, \beta \leq 1$), statistical convergence of order (α, β), double density by moduli.

Ideal, filter, admissible ideal, maximal admissible ideal, I-convergence, I^*-convergence, I-boundedness, I-limit point, I-cluster point, I- $limsup\ x_n$, I- $liminf\ x_n$, I-Cauchy sequences, I-statistical and I-lacunary statistical convergence, I-statistical and I-lacunary statistical convergence of order α (where $0 < \alpha \leq 1$), I-statistically pre-Cauchy sequences, I and I^*-convergence of double sequences.

References:

1. Hemen Dutta and Billy E. Rhoades, Current Topics in Summability Theory and Applications, Springer (2016)
2. P.N. Natarajan, Classical Summability Theory, Springer (2017)
3. *Johan Boos and Peter Cass, Classical and Modern Methods in Summability, OXFORD University Press, (2000)*

M5: Topics in Analytic Number Theory **Full Marks: 50 Credit: 2**

Dirichlet series, Multiplication of Dirichlet series, Euler products.
Riemann zeta function, functional equation, zero free regions, analytic continuation.
Dirichlet L-function, Dirichlet's Theorem for primes in an arithmetic progression. Functional equation and Euler product for L-functions.
Chebyshev's ψ function, Chebyshev's θ function, Analytic proof of Prime Number Theorem.

References:

1. *T.M. Apostol, Introduction to Analytic number theory, Springer-Verlag (1976).*
2. *Harold Davenport, Multiplicative Number Theory, Springer.*
3. *S. Lang, Algebric Number Theory, Addison-Wesley, 1970.*
4. *M. Ram Murty, Problems in analytic number theory, springer.*
5. *M. Ram Murty, Michael Dewar and Hester Graves, Problems in the theory of modular forms, Hindustan Book Agency.*
6. *Neal Koblitz, Introduction to Elliptic curves and Modular forms, 2nd Ed., springer.*
7. *E. C. Titchmarsh, 2nd edition revised by D. R. Heath-Brown, The Theory of Riemann Zeta function, Oxford Science Publications.*

8. *K. Chandrasekharan, "Introduction to Analytic Number Theory", Springer-Verlag, 1968.*
9. *H. Iwaniec, E. Kowalski, "Analytic Number Theory", American Mathematical Society Colloquium Publications 53, American Mathematical Society, 2004.*

M6: Topological Indices of Graphs **Full Marks: 50 Credit: 2**

Topological graph, History of topological indices, Topological indices of graphs, Degree based and Distance based topological index, Wiener index, Zagreb index, Shannon Wiener index, F-index, Gutman index, Application of topological indices.

References:

1. *Topological indices and related descriptors in QSAR and QSPR, Gordon and Breach, Amsterdam, The Netherlands, 1999.*
2. *Mathematical concepts in organic chemistry (Springer- Verlag), Berlin, 1986.*

M7: Advanced Modern Algebra **Full Marks: 50 Credit: 2**

Categories, Functors, Natural Transformations, Universal Properties, Representable Functors, Yoneda's Lemma, Limits, Colimits, Adjoint Functors.

Group-Rings, Semisimple rings and modules.

Representation theory of finite groups.

References:

1. *D.S. Dummit and R.M. Foote, Abstract Algebra (3e), John Wiley and Sons (Asian reprint).*
2. *J.J. Rotman, Advanced Modern Algebra, Springer (Indian reprint 2016).*
3. *N. Jacobson, Basic Algebra II, Dover Publication, 2018.*
4. *M.F. Atiyah and I.G. MacDonald, Introduction to commutative Algebra, CRC Press, 2019.*
5. *D. Eisenbud, Commutative Algebra, Springer, USA, 2004.*
6. *W.A. Adkins and S. H. Weintraub, Algebra, Springer-Verlag, 1999.*

M8: Finite and Boundary Element Methods **Full Marks: 50 Credit: 2**

Finite Element Method: Introduction to Finite Elements, FEM applied to discrete systems, Method of weighted residuals, FEM applied to one-dimensional linear static problems, FEM applied to two-dimensional linear static problems – scalar and vector field problems, introduction to dynamic problems.

Boundary Element Methods: Green's identity - Integral equation for the potential on the boundary, The Neumann and Dirichlet boundary conditions, Principle of boundary element method; conversion of basic weighted residue statement into boundary integral equation inverse statement, concept of fundamental solution; application to potential problem in two and three dimensions; type of boundary elements; Green's theorem - The Galerkin vector/"concentrated load" function, Somigliana's identity, The displacement and the traction boundary conditions, Direct vs. indirect formulations, Discretization of boundary into panels, Approximation of singularity distributions on the boundary, Galerkin vs. collocation approach, Matrix of influence coefficients, Evaluation of influence coefficients - The self-influence coefficient, Low-order (constant) vs. high-order (linear, quadratic) methods, Boundary shape discontinuities (corners), Applications.

<u>***References:***</u>

1. *C Pozrikidis, A Practical Guide to Boundary Element Methods with the Software Library BEMLIB, CHAPMAN & HALL/CRC, 2002.*
2. *J. T. Katsikadelis, Boundary Elements Theory and Applications, Elsevier, Oxford, 2002.*
3. *G. C. Hsiao, W. L. Wendland, Boundary Integral Equations, Springer-Verlag Berlin Heidelberg, 2008.*
4. *C Pozrikidis, Boundary Integral and Singularity Methods for Linearized Viscous Flow, Cambridge University Press, 1992.*

M9: Similarity Transformations and Perturbation Theory **Full Marks: 50 Credit: 2**

Prerequisite: Ordinary and Partial Differential Equations

General dimensional theory, Global similarity transformations, Transformation Groups, Infinitesimal Transformations, Invariant Functions, Prolongation and Invariance of Differential Equations (ODE and PDE), Invariant Solutions (Similarity Solutions). Parameter Perturbations, Coordinate Perturbations, Order Symbols and Gauge Functions, Asymptotic Expansions and Sequences, Straightforward expansions and sources of non-uniformity, Type change of a PDE, Method of Strained Coordinates, Method of matched and composite asymptotic expansions, Variation of parameters, Method of Multiple Scales.

<u>***References:***</u>

1. *Nayfeh, Ali Hasan, Introduction to perturbation techniques, John Wiley & Sons, 2011.*
2. *Murdock, James A, Perturbations: theory and methods, SIAM, 1999.*
3. *Bellman, Richard Ernest, Perturbation techniques in mathematics, engineering and physics, Courier Corporation, 2003.*
4. *Bluman, George W and Cole, Julian D, Similarity methods for differential equations, Springer Science & Business Media, 2012.*

M10: Advanced Numerical Techniques and MATLAB **Full Marks: 50 Credit: 2**

Prerequisite: Numerical Analysis, ODE, PDE

Revision of IVP and BVP, Single-step and Multi-step methods. System of first order ODE, higher order IVPs. Numerical solutions of BVP - Linear BVP, finite difference methods, shooting methods, stability, error and convergence analysis, nonlinear BVP, higher order BVP.

System of linear equations and eigenvalue problem: Operational counts for direct methods of solving system linear algebraic equations. Gaussian operational count for inversion of a matrix. Eigen value problem. General iterative method. Jacobi and Gauss. Seidel method. Relaxation method. Necessary and sufficient conditions for convergence. Speed of convergence. S.O.R. and S.U.R. methods. Gerschgorin’s circle theorem. Determination of eigenvalue by iterative methods, Ill conditioned system.

System of non-linear equations: Newton’s method. Existence of roots. Stability and convergence under variation of initial approximations. General iterative method for the system: x = g(x) and its sufficient condition for convergence. The method of steepest descent.

Finite difference method: Grids, Finite-difference approximations to derivatives. Linear Transport Equation: Upwind, Lax-Wendroff and Lax-Friedrich schemes, Von Neumann stability analysis, CFL condition, Lax-Richtmyer equivalence theorem, Solution of partial differential equations by finite difference method. Partial difference quotients. Discretization error. Idea of convergence and stability. Explicit and Crank Nicolson implicit method of solution of one-dimensional heat conduction equation: convergence and stability.

Standard and diagonal five point formula for solving Laplace and Poission equations. Explicit and Implicit method of solving Cauchy problem of one-dimensional wave equation, difference approximations in polar coordinates.

Finite Volume Methods: Basics and applications in solution of PDE.

Fundamentals of MATLAB, syntax, basic mathematical operations, Implementation of numerical schemes using MATLAB.

References:

1. *M. K. Jain, S.R.K. Iyenger and R.K. Jain, Computational Methods for Partial Differential Equations (Second edition), New Age International Publication (P) Ltd (2016)***.***
2. *K.W Mortons and D. F. Mayers, Numerical solution of partial differential equations (Second edition), Cambridge University press.*
3. *VitorianoRuas, Numerical Methods for Partial Differential Equations: An Introduction, Wiley (2016).*
4. *J. Kiusalaas, Numerical Methods in Engineering with MATLAB, Cambridge University Press, 2005.*
5. *M.K.Jain, Numerical solution of differential equations.*
6. *G.D.Smith, Numerical solution of partial differential equations.*
7. *S. R. Otto, James P. Denier, An Introduction To Programming And Numerical Methods In Matlab, Springer, 2005.*

M11: Modern Theory of Partial Differential Equations **Full Marks: 50 Credit: 2**

Prerequisites: Functional Analysis and Partial Differential Equations

Theory of distributions: supports, test functions, regular and singular distributions, generalized derivatives. Sobolev Spaces: definition and basic properties, approximation by smooth functions, dual spaces, trace and imbedding results. Elliptic Boundary Value Problems: abstract variational problems, Lax-Milgram Lemma, weak solutions and wellposedness with examples, regularityresult, maximum principles, eigenvalue problems. Semigroup Theory and Applications:exponential map, C0-semigroups, HilleYosida and Lummer-Phillips theorems, applications to heat and wave equations.

References:

1. *S. Kesavan, Topics in functional analysis and applications, (Wiley eastern, 1989).*
2. *L. C. Evans, Partial Differential Equations (second edition), AMS, Berkeley, 2010*
3. *M. Renardy, R. C. Rogers, An Introduction to Partial Differential Equations, Springer, 2004.*
4. *H. Brezis, Functional analysis, Sobolev spaces and Partial differential equations, Springer, 2011.*

M12: Advanced Trends in Continuum Mechanics **Full Marks: 50 Credit: 2**

Prerequisites: Continuum Mechanics

Nonlinear Elasticity: Preliminary Considerations, The Equilibrium Problem, Equilibrium Boundary Problems, Variational Formulation of Equilibrium, Isotropic Elastic Materials, Homogeneous Deformations, Homothetic Deformation, Extension of a Rectangular Block, Shear of a Rectangular Block, Universal Static Solutions, Constitutive Equations in Nonlinear Elasticity. Nondimensional Analysis of Equilibrium, Signorini's Perturbation Method for Mixed Problems, Signorini's Method for Traction Problems, Second-Order Hyperelasticity, Application of Signorini's Method.

Micropolar Elasticity: Preliminary Considerations, Kinematics of a Micropolar Continuum, Mechanical Balance Equations, Energy and Entropy, Elastic Micropolar Systems, The Objectivity Principle, Some Remarks on Boundary Value Problems, Asymmetric Elasticity.

Phase Equilibrium: Boundary Value Problems in Phase Equilibrium, Some Phenomenological Results of Changes in State, Equilibrium of Fluid Phases with a Planar and Spherical Interface Variational Formulation of Phase Equilibrium, Phase Equilibrium in Crystals, Wulff's Construction.

Introduction to Mixture Theory: Balance Laws, Classical Mixtures, Nonclassical Mixtures, Balance Equations of Binary Fluid Mixtures, Constitutive Equations, Phase Equilibrium and Gibbs' Principle.

Electromagnetism in Matter: Integral Balance Laws, Electromagnetic Fields in Rigid Bodies at Rest, Constitutive Equations for Isotropic Rigid Bodies, Approximate Constitutive Equations for Isotropic Bodies, Maxwell's Equations and the Principle of Relativity, Quasi-electrostatic and Quasi-magnetostatic Approximations, Balance Equations for Quasi-electrostatics, Isotropic and Anisotropic Constitutive Equations, Polarization Fields and the Equations of Quasi-electrostatics, More General Constitutive Equations.

Introduction to Magnetofluid Dynamics: An Evolution Equation for the Magnetic Field, Balance Equations in Magnetofluid Dynamics, Equivalent Form of the Balance Equations, Constitutive Equations, Ordinary Waves in Magnetofluid Dynamics, Alfven's Theorems, Laminar Motion Between Two Parallel Plates, Law of Iso-rotation.

Relativistic Continuous Systems: Lorentz Transformations, The Principle of Relativity, Minkowski Spacetime, Four-Dimensional Equation of Motion, Integral Balance Laws, The Momentum‑Energy Tensor, Fermi and Fermi‑Walker Transport, The Space Projector, Intrinsic Deformation Gradient, Relativistic Dissipation Inequality, Thermoelastic Materials in Relativity, Physical Meanings of Relative Quantities, Maxwell's Equation in Matter, Minkowski's Description, Amper's Model.

References:

1. *J. N. Reddy, An Introduction to Continuum Mechanics (Second edition), Cambridge University Press.*
2. *D. Rubin, E. Krempl, and W. Michael Lai, Introduction to Continuum Mechanics (Third edition), Pergamon press (1993).*
3. *Peter Chadwick, Continuum Mechanics: Concise Theory and Problems (Second Edition), Dover Publication Inc. (1999).*
4. *John W. Rudnicki, Fundamentals of Continuum Mechanics, John Wiley & Sons (2014).*
5. *A. J. M. Spencer, Continuum mechanics, Dover Publication Inc. (2004).*

M13: Advanced Fluid Mechanics **Full Marks: 50 Credit: 2**

Prerequisite: Ordinary and Partial differential equations, Vector and Tensor Calculus, Fluid Mechanics

Recapitulation: Concept of continuum and definition of a fluid. Body and surface forces, stress tensor.Scalar and vector fields, Eulerian and Lagrangian description of flow. Motion of fluid element; translation, rotation and vorticity; strain rate tensor, continuity equation, stream function and velocity potential.

Constitutive equations, derivation of Navier-Stokes equations. Exact solutions of Navier-Stokes equations: plane Poiseuille flow and Couette flow, Hagen-Poiseuille flow, flow between two concertic rotating cylinders, Stokes first and second problem, Hiemenz flow, flow near a rotating disk, flow in convergent-divergent channels. Slow viscous flow: Stokes and Oseen's approximation, theory of hydrodynamic lubrication. Thin-film equations. Boundary layer: derivation, exact solutions, Blasius, Falkner Skanseries solution and numerical solutions. Approximate methods. Momentum integral method. Two dimensional and axisymmetric jets.

Introduction to Hydrodynamic stability: linear stability of plane Poiseuille flow, Orr-Sommerfeld equation. Description of turbulent flow, velocity correlations, Reynolds stresses, Prandtl s Mixing Length Theory, Karman s velocity defect law, universal velocity distribution. Concepts of closure model, eddy viscosity models of turbulence- zero equation, one equation and two-equation models.

References:

1. *Frank. M. White, Fluid Mechanics, McGraw-Hill Higher Education, 8th edition.*
2. *I. Kohen and P. K. Kundu, Fluid Mechanics, Elsevier, 3rd edition.*
3. *Frank M. White, Viscous Fluid Flow, McGraw-Hill Higher Education, 3rd edition*
4. *George K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press.*
5. *S. K. Som, G. Biswas and S. Chakraborty, Introduction to Fluid Mechanics and Fluid Machines, Tata-McGraw-Hill, 3rd Edition.*
6. *S. W. Yuan, Foundation of Fluid Mechanics, Prentice-Hall, 1967.*

M14: Mathematical Methods **Full Marks: 50 Credit: 2**

Two-point boundary-value problems, Green's functions, Construction of Green's functions, Nonhomogeneous boundary conditions, Sturm-Liouville Systems, Eigen values and Eigen functions, Eigen function expansions and completeness; Hypergeometric equation and functions, Properties of hypergeometric functions, Legendre equation and Legendre polynomial, Generating function for Legendre polynomial, Recurrence relations between Legendre polynomials, Rodrigueâs formula. Orthogonality of Legendre polynomial, Associated Legendre equation and Legendre function, Bessel equation and its solution, Bessel functions, Modified Bessel function, Generating function for Bessel function, Recurrence relations between Bessel functions, Orthogonality of Bessel functions.; Autonomous systems, Stability for Linear systems with constant coefficients, Linear plane autonomous systems, perturbed systems, Method of Lyapunov for nonlinear systems. Limit cycles of Poincare; Coordinate transformations, Definition of Tensors, Summation convention, Kronecker Delta, Covariant, contravariant and mixed tensors. Fundamental operations with tensors, the line element and metric tensor, length of a vector, Christoffelâs symbols, the covariant derivative, tensor form of gradient, divergence and curl. Examples from continuum mechanics, elasticity, plasticity, fluids.

References:

1. *Arfken, George B and Weber, Hans J, Mathematical methods for physicists, American Association of Physics Teachers, 1999.*
2. *Ken F. Riley, Mike P. Hobson, Stephen J. Bence, Mathematical Methods for Physics and Engineering, Cambridge, 2018.*
3. *Samuel D Lindenbaum, Mathematical Methods in Physics, World Scientific, 1996.*
4. *H. W. Wyld, Mathematical Methods for Physics, CRC Press, 2018.*

M15: Optimization Techniques **Full Marks: 50 Credit: 2**

Stochastic programming, chance constrained programming and two-stage programming, geometric programming, polynomial and signomial programming, dual based methods, primal based methods, Dynamic programming, single stage and multi-stage programming, forward and backward process, deterministic and probabilistic dynamic programming models, interior point methods, projective and scaling methods for linear programming.

References:

1. *C. B Gupta, Optimization Techniques in Operation Research, I.K. International Publishing House Pvt. Limited, 2008.*
2. *Xin-She Yang, Optimization Techniques and Applications with Examples, Wiley, 2018.*
3. *Cornelius T. Leondes, Optimization Techniques, Elsevier Science, 1998.*
4. *Chander Mohan, Kusum Deep, Optimization Techniques, New Age Science, 2009.*
5. *Sukanta Nayak, Fundamentals of Optimization Techniques with Algorithms, Elsevier Science, 2020.*

M16: Decision Theory **Full Marks: 50 Credit: 2**

Games and statistical games, statistical decision problem, decision function, risk function, prior and posterior distribution, Bayes risk and Bayes rules, least favourable prior, minimaxity, admissibility and complete classes, admissibility of Bayes rules, existence of minimal complete class and Bayes rules, the supporting and separating hyperplane theorems, essential completeness of the class of nonrandomized rules, minimax and complete class theorems, solving for minimax rules, essential completeness of class of rules based on sufficient statistics, continuity of risk functions, invariant decision problems, admissible and minimax invariant decision rules.

References:

1. *Martin Peterson, An Introduction to Decision Theory, Cambridge University Press, 2017.*
2. *Herman Chernoff, Lincoln E. Moses, Elementary Decision Theory, Dover Publications, 2012.*
3. *Douglas John White, Decision Theory, Transaction Publishers, 1969.*
4. *Giovanni Parmigiani, Lurdes Inoue, Decision Theory Principles and Approaches, Wiley, 2009.*

M17: Differential Geometry of Manifolds **Full Marks: 50 Credit: 2**

Differentiable manifolds: Definition and Examples of Topological manifolds, smooth maps and diffeomorphisms, Definition and examples of differentiable manifolds, derivatives of smooth maps, local expression for the differential, curves in a manifold, immersion and submersion, rank, critical and regular points, submanifolds and regular submanifolds. Tangent and Cotangent spaces, Tangential maps, Vector Fields on smooth manifolds, Lie Brackets and its properties, Integral Curves and Flows, f-related vector fields, 1-parameter group of transformations, Differential forms, local expression for a k-form, pull back of a k-form, wedge product, Exterior algebra and Exterior derivatives.

Riemannian manifolds: Affine connections, Riemannian and semi-Riemannian metrics, Riemannian connection, Riemann curvature tensor, Ricci tensor, Scalar curvature, Sectional Curvature, Semi-symmetric and quarter symmetric metric connections on Riemannian manifolds, Einstein manifolds and its generalizations, Manifolds of constant curvature and its generalizations, Some transformations (e.g., Conformal transformation, Projective transformation, Concircular transformation, Conharmonic transformation) on Riemannian manifolds, Locally symmetric Riemannian manifolds due to Cartan and its generalizations, Product manifolds, Warped product manifolds.

Submanifolds: Embedded Submanifolds, Immersed Submanifolds, Hypersurfaces of Riemannian manifolds, Induced connection and second fundamental form, Gauss and Weingarten formulae, Equations of Gauss, Codazzi and Ricci, Mean curvature, Totally geodesic and totally umbilical submanifolds, Minimal submanifolds.

References:

1. *W. M. Boothby; An Introduction to Differentiable Manifolds and Riemanian Geometry; Academic Press, Revised, 2003.*
2. *L. Conlon; Differentiable Manifolds, A First Course; Birkhauser (Second Edition), 2008.*
3. *W. D. Curtis and F. R. Miller; Differential Manifolds and Theoretical Physics; Academic Press, 1985.*
4. *S. Helgason; Differential Geometry, Lie Groups and Symmetric Spaces; Academic Press, 1978.*

5. *N.J. Hicks; Notes on Differential Geometry; Notes.*
6. *Kobayashi & Nomizu; Foundations of Differential Geometry, Vol-I; Interscience Publishers, 1963.*
7. *S. Kumaresan; A course in Differential Geometry and Lie-groups; Hindustan Book Agency.*
8. *S. Lang; Differential and Riemannian manifolds; Springer-Verlag, 1995.*
9. *John M. Lee; Introduction to smooth manifolds; Springer.*
10. *M. Spivak; A Comprehensive Introduction to Differential Geometry, volumes 1 and 2; Publish or Perish, 1979*
11. *K. Yano and M. Kon; Structure on Manifolds; World Scientific, 1984*
12. *T. J. Willmore, Riemannian Geometry, Oxford University Press, 1997.*
13. *K. Yano and M. Kon, Structure on Manifold, World Scientific Publication, Singapore, 1984.*
14. *J. M. Lee, Riemannian Manifolds, An Introduction to Curvature, Springer-Verlag, 2005.*
15. *B. Y. Chen, Geometry of Submanifolds, Marcel Dekker. Inc., New York, 1973.*

M18: Mathematical Theory of Elasticity **Full Marks: 50 Credit: 2**

Prerequisite: concept of tensor

Analysis of strain: Introduction, affine transformation, infinitesimal affine deformation, a geometrical interpretation of the components of strain, strain quadric of Cauchy, principal strains, invariants, general infinitesimal deformation, equations of compatibility, finite deformations.

Analysis of stress: Body and surface forces, stress tensor, equations of equilibrium, transformations of coordinates, stress quadric of Cauchy, maximum normal and shear stresses.

Stress-strain relations: Hooke's law, Generalized Hooke's law, Homogeneous and isotropic bodies, elastic moduli of isotropic bodies, elastic moduli of isotropic bodies, equilibrium equations for an isotropic elastic solid, dynamical equations of an isotropic elastic solid, the strain-energy function and its connection with Hooke's law, uniqueness of solution of the boundary-value problems of Elasticity, Saint-Venant's principle.

Waves in elastic media: Body waves of dilatation and distortion. Surface waves-Rayleigh and love waves.

Thermoelasticity: Introduction to Thermoelasticity, Basic equations of thermoelasticity, hyperbolic thermoelasticity.

References:

1. *I. S. Sokolnikoff, Mathematical theory of elasticity.*
2. *A. E. H. Love, A Treatise on the Mathematical Theory of Elasticity.*