UNIVERSITY OF NORTH BENGAL

Syllabus for

M.Phil.

in

MATHEMATICS

Department of Mathematics, University of North Bengal, Raja Ram Mohunpur, P.O.- N.B.U., Dist- Darjeeling, West Bengal, India, Pin-734013

S. Ghosal
18/12/2020

Signature of HOD
Department of Mathematics, NBU

HEAD
Department of Mathematics
University of North Bengal
# Structure of Syllabus for M.Phil. in Mathematics

## Semester-I

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Course Type</th>
<th>Course Name</th>
<th>Full Marks (External)</th>
<th>Full Marks (Practical)</th>
<th>Full Marks (Internal)</th>
<th>Full Marks (Total)</th>
<th>Credit</th>
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<tbody>
<tr>
<td>MPHILMA-101</td>
<td>Theory</td>
<td>Research Methodology: Research Foundation</td>
<td>20</td>
<td></td>
<td>5</td>
<td>25</td>
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<tr>
<td>MPHILMA-102</td>
<td>Practical</td>
<td>Research Methodology: Computer Application in Research</td>
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<tr>
<td>MPHILMA-103</td>
<td>Theory</td>
<td>Elective: Any two papers to be chosen from <strong>Table-I</strong>, based on research interest of the students and availability of suitable teachers/Supervisors.</td>
<td>40 + 40</td>
<td>10 + 10</td>
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<td>50 + 50 + 50 + 50</td>
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Total: 120 30 150 12

## Semester-II

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<th>Course Code</th>
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<th>Full Marks (Practical)</th>
<th>Full Marks (Internal)</th>
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<tbody>
<tr>
<td>MPHILMA-201</td>
<td>Theory</td>
<td>Elective: Any three papers to be chosen from <strong>Table-II</strong>, based on research interest of the students and availability of suitable teachers/Supervisors.</td>
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<td>10 + 10</td>
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Total: 120 30 150 12
### Semester-III

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<tr>
<th>Course Code</th>
<th>Course Type</th>
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<th>Full Marks (Practical)</th>
<th>Full Marks (Internal)</th>
<th>Full Marks (Total)</th>
<th>Credit</th>
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<td>Preliminary Dissertation</td>
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Table: I

<table>
<thead>
<tr>
<th>Elective Paper Sub-Code</th>
<th>Title of the Paper</th>
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<tbody>
<tr>
<td>M1</td>
<td>Theory of Convergence and Topological Hyperalgebra</td>
</tr>
<tr>
<td>M2</td>
<td>Basic Theory of Entire and Meromorphic Functions</td>
</tr>
<tr>
<td>M3</td>
<td>Measure Theory</td>
</tr>
<tr>
<td>M4</td>
<td>Summability Methods-I</td>
</tr>
<tr>
<td>M5</td>
<td>Topics in Analytic Number Theory</td>
</tr>
<tr>
<td>M6</td>
<td>Topological Indices of Graphs</td>
</tr>
<tr>
<td>M7</td>
<td>Advanced Modern Algebra-I</td>
</tr>
<tr>
<td>M8</td>
<td>Finite and Boundary Element Methods</td>
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<tr>
<td>M9</td>
<td>Similarity Transformations and Perturbation Theory</td>
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<tr>
<td>M10</td>
<td>Advanced Numerical Techniques and MATLAB</td>
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<td>M11</td>
<td>Modern Theory of Partial Differential Equations</td>
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<td>M12</td>
<td>Advanced Trends in Continuum Mechanics</td>
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<td>M13</td>
<td>Advanced Fluid Mechanics</td>
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<td>M14</td>
<td>Mathematical Methods</td>
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<td>M15</td>
<td>Optimization Techniques</td>
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<td>M16</td>
<td>Decision Theory</td>
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<td>M17</td>
<td>Differential Geometry of Manifolds</td>
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<td>M18</td>
<td>Mathematical Theory of Elasticity</td>
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<td>Elective Paper Sub-Code</td>
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<tr>
<td>N1</td>
<td>General Theory of Integration</td>
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<tr>
<td>N2</td>
<td>Value Distribution Theory of Meromorphic Functions</td>
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<tr>
<td>N3</td>
<td>Summability Methods-II</td>
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<td>N4</td>
<td>Advanced Modern Algebra-II</td>
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<td>N5</td>
<td>Modular Forms</td>
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<td>N6</td>
<td>Complex Manifolds</td>
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<td>N7</td>
<td>Contact Manifolds</td>
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<td>N8</td>
<td>General Theory of Relativity and Cosmology</td>
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<td>N9</td>
<td>Advanced Quantum Mechanics</td>
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<td>N10</td>
<td>Boundary Layer Theory and Turbulence</td>
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<td>N11</td>
<td>Chaos Theory and Fractals</td>
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<td>N12</td>
<td>Non-linear Partial Differential Equations</td>
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<td>N13</td>
<td>Fuzzy Sets and Applications</td>
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<td>N14</td>
<td>Mathematical Logic</td>
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<td>N15</td>
<td>Multivariate Analysis</td>
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<td>N16</td>
<td>Numerical Optimization</td>
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<td>N17</td>
<td>Lie Groups and Lie Algebras</td>
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<tr>
<td>N18</td>
<td>Queuing Theory</td>
</tr>
<tr>
<td>N19</td>
<td>Data Structure and Algorithm</td>
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**Detailed Syllabus**

**Semester-I**

**MPHILMA-101: Research Methodology: Research Foundation**  
**Full Marks: 25 Credit: 2**

**Introduction**- Meaning, purpose, objectives, characteristics, motivation, significance, types of research; approaches, process, methods and methodology used in research; criteria of good research; research methods in general and Mathematical Sciences in particular.

**The Research Problem** – Research problems and sub-problems identification, stating, defining, techniques involved in defining problem.

**Interpretation and Report Writing**- Techniques of interpretation- significance, types, steps, checklist/precautions and characteristics of research documentation i.e. reviews, treatise, monographs, abstracts, articles, technical reports, white papers, research papers, thesis etc.- issues and techniques of writing project proposals, paper presentation and soft skills.

**Ethical issues and Professional Conduct**- Ethics in general, Professional Ethics, ethical Issues and their significance those arise from Computer Technology, General Moral Imperatives, Concepts and issues related to plagiarism and Intellectual Property Rights

**Literature Review** – Importance of Literature review in defining a problem, including literature in research proposal, critique, survey & peer review process, identifying gap areas from literature review; Major Research areas, Journals, Publication, Conferences and Status of Research in the field of Mathematical Sciences.

Extensive review work has to be undertaken by the candidate in relevance to his/her topic of research interest. The candidate will be required to submit a written report on the survey to be evaluated by the Departmental Council. Candidate will also be required to deliver a seminar lecture on the survey work.

**References:**

6. *Conducting Research Literature Reviews: From the Internet to Paper* by Fink, A., Sage Publications  
9. *Research Methodology* by Dalip Kumar Bhattacharya  
10. *Research Methodology* by C.H Chaudhary, RBSA Publication  
11. *Statistical Techniques* by S.P. Gupta, Sultan Chand & Sons  
13. WWW(Web Sources)
MPHILMA-102: Research Methodology: Computer Application in Research

Full Marks: 25 Credit: 2

Historical background of LATEX and its pronunciation:
Creating a new document, Opening and saving a Document, Use of document classes like article.cls, amsart.cls, books.cls, report.cls; Adding packages like amssymb, amsmath, amsthm, amsfonts, graphics, graphix, times;

Entering Text, Editing Text


Formatting Your Document: Formatting with tags: theorem, definition, corollary, lemma, proposition, example, acknowledgement, axiom, proposition and changing their styles; Formatting the Page, Changing body text point size, paper size, orientation, title page, two column category, equation number position; Use of unit of measurement: ex, em, pt, cm, mm, in; Definition of commands : \newcommand, \renewcommand, \setcounter, \addcounter, \newenvironment, \renewenvironment; Introduction to spacing: horizontal spacing, vertical spacing, small space, big space etc; Different pagebreaks: newline, line break; Making section, subsection etc, Alignment of texts, pictures, creating post script files (*.ps, *.eps) and inserting them into Tex files.

Saving Your Documents: Saving Portable LATEX Files, Exporting Files.

Previewing and Printing Your Document: Creating PDF Files, Exporting Documents as HTML Files.

Special features of Scientific Workplace: Computing and Plotting, Evaluate and Evaluate Numerically, Factor, Combine, Expand, Simplify, Equality, Solve Exact and Numeric, Work with Polynomials, 2-D and 3-D Plots, Compute in Place, Perform Matrix Operations, Solve Differential Equations, Compute Statistics, Compute with Units of Measure, Create Exams and Quizzes.


Basics of operating systems: handling different operating systems, Literature survey using web, handling search engines.

Computer usage for collecting/analyzing data: simulations using fortran/ C/ Mathematica/ MATLAB/Mathcad----Solving Differential equation, numerical solution of differential equation, Integration, Solution of Algebraic equation, numerical solution of differential equation, Integration, Solution of Algebraic equation; Plotting of solution curve of system of differential equations, Two- and three-dimensional plotting, plotting data, using options, and creating dynamic and interactive graphics.

Preparing presentations:

i) Research papers: Using word processing software – MS Word/LaTeX/others, Drawing graphs and diagrams – Origin/Xmgrace/Excel/others.

ii) Seminar presentations – Beamer/Power point for oral and poster presentations.
Use of Acrobat reader, Texnic Centre, WinEdit, Ghost View; Conversion of graphics in different formats by Ghost View.

References:

2. Getting Started with Scientific Workplace®, Scientific Word® & Scientific Notebook® Version 5- Susan Bagby, Publisher: MacKichan Software
3. The Computer Science of TeX and LaTeX by Victor Eijkhout
Theory of Convergence

Topological Hyperalgebra:
Introduction to Topological Groups and Semigroups:
Semitopological groups: The concept of a semitopological group, neighborhood system of identity, constructions of new semitopological groups from old, locally compact semitopological groups. Topological group: Translation in topological groups, neighborhood system of identity, separation axioms in topological groups, uniform structure on a topological group, sub groups, quotient groups, locally compact groups, continuous and open homomorphism, open homomorphism and closed graph theorems.

Introduction to Topological Rings:
Definition, examples of topological rings, topological modules, vector space, and algebras, neighborhoods of zero, subrings, ideals, submodules.

Metrizable groups, completions of commutative Hausdorff groups, completions of topological ring and modules, locally bounded modules and rings.

Introduction to algebraic hyperstructures:
What algebraic hyperstructures are? A historical development of algebraic hyperstructures. The hypergroup of Marty: Definitions, examples, subhypergroups, Some kinds of subhypergroups, homomorphism of hypergroups, Join spaces, Canonical hypergroups, Polygroups, regular and strongly regular relations, the fundamental relation, complete parts, the heart of a hypergroup, complete semihypergroups, complete hypergroups, Polygroups: Definition, examples, extension of polygroups by polygroups, subpolygroups and quotient polygroups, isomorphism theorems of polygroups, generalized permutation, permutation polygroups, representation of polygroups, solvable polygroups, nilpotent polygroups. The hyperring of Krasner: Definition and constructions of Krasner hyperrings, hyperideals, quotient hyperrings and homomorphisms, special hypeideals.
Introduction to topological algebraic hyperstructures:

Topological Hypergroups in the sense of Marty: Definitions, examples, topological subhypergroup, compact Hausdorff topological hypergroup.

Topological Polygroups: Definition, examples, results, subpolygroups of a topological polygroup, isomorphism theorems, rolls of complete parts in topological polygroups, left big subsets of topological polygroups.

Topological Polygroups: Definition, examples, related identity, some results.

References:


M2: Basic Theory of Entire and Meromorphic Functions  

Full Marks: 50 Credit: 4

Elementary theory of entire and meromorphic functions, Poisson –Jensen formula, Nevanlinna characteristic function, Nevanlinna’s first fundamental theorem, Cartan’s identity, Orders of growth, Comparative growth of T(r) and log M(r), Weierstrass product, Representation of a meromorphic function in terms of its zeros and poles, Nevanlinna’s second fundamental theorem, Various types of deficiencies of meromorphic functions and their properties.

References:

M3: Measure Theory

Lebesgue Measure space on the Euclidean space. Lebesgue Outer Measure on the Euclidean space. Definition and properties. Regularity properties; Borel measure space of n–dimension; Completion of Borel Measure space; Estimation of Lebesgue measure on $\mathbb{R}^n$ by closed and compact sets and approximation by open sets.

Hausdorff Measures, Regularity and Hausdorff dimension, Hausdorff measures of integral and functional dimensions.

References:
1. Lectures on Real Analysis by J. Yeh, World Scientific.

M4: Summability Methods-I

Density, asymptotic density/ natural density, statistical convergence, statistical boundedness, statistical limit point, statistical cluster point, relation between statistical limit points set and statistical cluster points set, statistical limit superior and limit inferior, lacunary statistical convergence, strongly Cesaro summability, natural density of order $\alpha$ (where $0 < \alpha \leq 1$), statistical convergence of order $\alpha$, $\lambda$-statistical convergence, strong $(V,\lambda)$-summability, $ab\beta$-statistical convergence, density by moduli, $f$-statistical convergence, $f$-statistical limit point, $f$-statistical cluster point, relation between $f$-statistical limit points set and $f$-statistical cluster points set, weighted statistical convergence, statistical convergence in probability.

Double natural density, statistical convergence of double sequences, statistical Cauchy sequence, statistical convergence of multiple sequences, relation between statistical convergence and strongly Cesaro summable sequences, statistical limit points of double sequences, double density of order $(\alpha, \beta)$ (where $0 < \alpha, \beta \leq 1$), statistical convergence of order $(\alpha, \beta)$, double density by moduli.

Ideal, filter, admissible ideal, maximal admissible ideal, $I$-convergence, $I^*$-convergence, $I$-boundedness, $I$-limit point, $I$-cluster point, $I$-limsup $x_n$, $I$-liminf $x_n$, $I$-Cauchy sequences, $I$-statistical and $I$-lacunary statistical convergence, $I$-statistical and $I$-lacunary statistical convergence of order $\alpha$ (where $0 < \alpha \leq 1$), $I$-statistically pre-Cauchy sequences, $I$ and $I^*$-convergence of double sequences.

References:

M5: Topics in Analytic Number Theory

Dirichlet series, Multiplication of Dirichlet series, Euler products.
Riemann zeta function, functional equation, zero free regions, analytic continuation.
Dirichlet L-function, Dirichlet’s Theorem for primes in an arithmetic progression. Functional equation and Euler product for L-functions.
Chebyshev’s $\psi$ function, Chebyshev’s $\theta$ function, Analytic proof of Prime Number Theorem.
References:


M6: Topological Indices of Graphs

Full Marks: 50 Credit: 4

Topological graph, History of topological indices, Topological indices of graphs, Degree based and Distance based topological index, Wiener index, Zagreb index, Shannon Wiener index, F-index, Gutman index, Application of topological indices.

References:


M7: Advanced Modern Algebra-I

Full Marks: 50 Credit: 4

Categories, Functors, Natural Transformations, Universal Properties, Representable Functors, Yoneda’s Lemma, Limits, Colimits, Adjoint Functors.
Group-Rings, Semisimple Rings and Modules.
Representation Theory of Finite Groups.

References:


M8: Finite and Boundary Element Methods

Full Marks: 50 Credit: 4

Finite Element Method: Introduction to Finite Elements, FEM applied to discrete systems, Method of weighted residuals, FEM applied to one-dimensional linear static problems, FEM applied to two-dimensional linear static problems – scalar and vector field problems, introduction to dynamic problems.
Boundary Element Methods: Green’s identity - Integral equation for the potential on the boundary, The Neumann and Dirichlet boundary conditions, Principle of boundary element method; conversion of basic weighted residue statement into boundary integral equation inverse statement, concept of fundamental solution; application to potential problem in two and three dimensions; type of boundary elements; Green’s theorem - The Galerkin vector/"concentrated load" function, Somigliana’s identity, The displacement and the traction boundary conditions, Direct vs. indirect formulations, Discretization of boundary into panels, Approximation of singularity distributions on the boundary, Galerkin vs. collocation approach, Matrix of influence coefficients, Evaluation of influence coefficients - The self-influence coefficient, Low-order (constant) vs. high-order (linear, quadratic) methods, Boundary shape discontinuities (corners), Applications.

References:


M9: Similarity Transformations and Perturbation Theory

Prerequisite: Ordinary and Partial Differential Equations

General dimensional theory, Global similarity transformations, Transformation Groups, Infinitesimal Transformations, Invariant Functions, Prolongation and Invariance of Differential Equations (ODE and PDE), Invariant Solutions (Similarity Solutions). Parameter Perturbations, Coordinate Perturbations, Order Symbols and Gauge Functions, Asymptotic Expansions and Sequences, Straightforward expansions and sources of non-uniformity, Type change of a PDE, Method of Strained Coordinates, Method of matched and composite asymptotic expansions, Variation of parameters, Method of Multiple Scales.

References:


M10: Advanced Numerical Techniques and MATLAB

Prerequisite: Numerical Analysis, ODE, PDE

Revision of IVP and BVP, Single-step and Multi-step methods. System of first order ODE, higher order IVPs. Numerical solutions of BVP - Linear BVP, finite difference methods, shooting methods, stability, error and convergence analysis, nonlinear BVP, higher order BVP.


Finite Volume Methods: Basics and applications in solution of PDE.

Fundamentals of MATLAB, syntax, basic mathematical operations, Implementation of numerical schemes using MATLAB.

References:


M11: Modern Theory of Partial Differential Equations

Prerequisites: Functional Analysis and Partial Differential Equations


References:

M12: Advanced Trends in Continuum Mechanics

Prerequisites: Continuum Mechanics


**Phase Equilibrium:** Boundary Value Problems in Phase Equilibrium, Some Phenomenological Results of Changes in State, Equilibrium of Fluid Phases with a Planar and Spherical Interface Variational Formulation of Phase Equilibrium, Phase Equilibrium in Crystals, Wulff’s Construction.


**Introduction to Magnetofluid Dynamics:** An Evolution Equation for the Magnetic Field, Balance Equations in Magnetofluid Dynamics, Equivalent Form of the Balance Equations, Constitutive Equations, Ordinary Waves in Magnetofluid Dynamics, Alfven’s Theorems, Laminar Motion Between Two Parallel Plates, Law of Iso-rotation.


**References:**

M13: Advanced Fluid Mechanics

Prerequisite: Ordinary and Partial differential equations, Vector and Tensor Calculus, Fluid Mechanics

Recapitulation: Concept of continuum and definition of a fluid. Body and surface forces, stress tensor, Scalar and vector fields, Eulerian and Lagrangian description of flow. Motion of fluid element; translation, rotation and vorticity; strain rate tensor, continuity equation, stream function and velocity potential.


References:


M14: Mathematical Methods

Two-point boundary-value problems, Green’s functions, Construction of Green’s functions, Nonhomogeneous boundary conditions, Sturm-Liouville Systems, Eigen values and Eigen functions, Eigenfunction expansions and completeness.; Hypergeometric equation and functions, Properties of hypergeometric functions, Legendre equation and Legendre polynomial, Generating function for Legendre polynomial, Recurrence relations between Legendre polynomials, Rodrigueâs formula. Orthogonality of Legendre polynomial, Associated Legendre equation and Legendre function, Bessel equation and its solution, Bessel functions, Modified Bessel function, Generating function for Bessel function, Recurrence relations between Bessel functions, Orthogonality of Bessel functions.; Autonomous systems, Stability for Linear systems with constant coefficients, Linear plane autonomous systems, perturbed systems, Method of Lyapunov for nonlinear systems. Limit cycles of Poincare.; Coordinate transformations, Definition of Tensors, Summation convention, Kronecker Delta, Covariant, contravariant and mixed tensors. Fundamental operations with tensors, the line element and metric tensor, length of a vector, Christoffelâs symbols, the covariant derivative, tensor form of gradient, divergence and curl. Examples from continuum mechanics, elasticity, plasticity, fluids.
References:


M15: Optimization Techniques

Stochastic programming, chance constrained programming and two-stage programming, geometric programming, polynomial and signomial programming, dual based methods, primal based methods, Dynamic programming, single stage and multi-stage programming, forward and backward process, deterministic and probabilistic dynamic programming models, interior point methods, projective and scaling methods for linear programming.

References:


M16: Decision Theory

Games and statistical games, statistical decision problem, decision function, risk function, prior and posterior distribution, Bayes risk and Bayes rules, least favourable prior, minimaxity, admissibility and complete classes, admissibility of Bayes rules, existence of minimal complete class and Bayes rules, the supporting and separating hyperplane theorems, essential completeness of the class of nonrandomized rules, minimax and complete class theorems, solving for minimax rules, essential completeness of class of rules based on sufficient statistics, continuity of risk functions, invariant decision problems, admissible and minimax invariant decision rules.

References:


M17: Differential Geometry of Manifolds

**Riemannian manifolds:** Affine connections, Riemannian and semi-Riemannian metrics, Riemannian connection, Riemann curvature tensor, Ricci tensor, Scalar curvature, Sectional Curvature, Semi-symmetric and quarter symmetric metric connections on Riemannian manifolds, Einstein manifolds and its generalizations, Manifolds of constant curvature and its generalizations, Some transformations (e.g., Conformal transformation, Projective transformation, Conicircular transformation, Conharmonic transformation) on Riemannian manifolds, Locally symmetric Riemannian manifolds due to Cartan and its generalizations, Product manifolds, Warped product manifolds.

**Submanifolds:** Embedded Submanifolds, Immersed Submanifolds, Hypersurfaces of Riemannian manifolds, Induced connection and second fundamental form, Gauss and Weingarten formulae, Equations of Gauss, Codazzi and Ricci, Mean curvature, Totally geodesic and totally umbilical submanifolds, Minimal submanifolds.

**References:**

2. L. Conlon; Differentiable Manifolds, A First Course; Birkhauser (Second Edition), 2008.
5. N.J. Hicks; Notes on Differential Geometry; Notes.
7. S. Kumaresan; A course in Differential Geometry and Lie-groups; Hindustan Book Agency.
8. S. Lang; Differential and Riemannian manifolds; Springer-Verlag, 1995.
9. John M. Lee; Introduction to smooth manifolds; Springer.
10. M. Spivak; A Comprehensive Introduction to Differential Geometry, volumes 1 and 2; Publish or Perish, 1979
11. K. Yano and M. Kon; Structure on Manifolds; World Scientific, 1984

**M18: Mathematical Theory of Elasticity**

Full Marks: 50 Credit: 4

Prerequisite: concept of tensor

Analysis of strain: Introduction, affine transformation, infinitesimal affine deformation, a geometrical interpretation of the components of strain, strain quadric of Cauchy, principal strains, invariants, general infinitesimal deformation, equations of compatibility, finite deformations.

Analysis of stress: Body and surface forces, stress tensor, equations of equilibrium, transformations of coordinates, stress quadric of Cauchy, maximum normal and shear stresses.

Stress-strain relations: Hooke’s law, Generalized Hooke’s law, Homogeneous and isotropic bodies, elastic moduli of isotropic bodies, elastic moduli of isotropic bodies, equilibrium equations for an isotropic elastic solid, dynamical equations of an isotropic elastic solid, the strain-energy function and its connection with Hooke’s law, uniqueness of solution of the boundary-value problems of Elasticity, Saint-Venant’s principle.

Waves in elastic media: Body waves of dilatation and distortion. Surface waves-Rayleigh and love waves.

Thermoelasticity: Introduction to Thermoelasticity, Basic equations of thermoelasticity, hyperbolic thermoelasticity.

**References:**

Semester-II

N1: General Theory of Integration  Full Marks: 50  Credit: 4


Reference:

1. A Modern Theory of Integration, R. G. Bartle, AMS

N2: Value Distribution Theory of Meromorphic Functions  Full Marks: 50 Credit: 4

Uniqueness of entire and meromorphic functions, Nevanlinna’s five value theorem, Distribution of the values of meromorphic functions and their derivatives, Milloux theorem, Exceptional values of meromorphic functions and their derivatives, Multiple value and uniqueness.

References:


N3: Summability Methods-II  Full Marks: 50 Credit: 4

Rough convergence, dependence on roughness degree, rough Cauchy sequences, rough limit set, core of a sequence, Chebyshev centers, rough statistical convergence, rough statistical cluster point, rough \( I \)-convergence, rough \( I \)-convergence of double sequences, rough weighted statistical convergence, rough weighted \( I \)-convergence, rough weighted \( I \)-cluster point, rough weighted \( I \)-lacunary statistical convergence, rough Wijsman convergence, asymptotic cones, characterization of uniform rotundity in every direction in terms of rough convergence, rough convergence in probability.

References:

N4: Advanced Modern Algebra-II


References:


N5: Modular Forms


References:


N6: Complex Manifolds

Riemannian manifolds, Affine Connections (Koszul), Torsion and Curvature tensor field on Affine Connection, Covariant Differential. Almost Complex Manifolds: Introduction, algebraic Preliminaries, Nijenhuis tensor, Eigen values of the complex structure, Existence theorem and Integrability condition of an almost complex structure, Contravariant and covariant almost analytic vector field, Almost Hermite Manifolds, curvature tensor, Holomorphic sectional curvature, Linear connection in an almost Hermite manifold, Kähler Manifold: Kähler Manifolds, Holomorphic sectional curvature, Bochner curvature tensor, Affine connection in Kähler manifolds, Conformally flat Kähler manifolds, Projective correspondence
between two Kähler manifolds, Nearly Kähler Manifolds, Para Kähler Manifolds, conformal flatness of para Kähler manifolds.

Submanifolds of Kähler manifolds: Kählerian submanifolds, Anti-invariant submanifolds of Kählerian manifolds, CR-submanifolds of Kählerian manifolds.

References:

1. R.S.Mishra; Structures on a Differentiable Manifold and Their Applications; Chandrama Prakashan, Allahabad, 1984.
5. R. C. Gunning; Lectures on Riemann Surfaces; Princeton, Princeton University Press, 1966

N7: Contact Manifolds

Full Marks: 50 Credit: 4

Contact manifold, contact metric manifold, almost contact manifold, Killing vector field, properties of $\phi$, the tensor field $h$, some curvature properties of contact metric manifold. K-contact Manifolds, Characterizations of K-contact manifolds, some curvature properties of K-contact manifolds, sectional curvature of K-contact manifolds, Locally symmetric and Ricci symmetric K-contact manifolds, semi-symmetric and Ricci-semisymmetric K-contact manifolds.

Sasakian manifolds: Introduction, some curvature properties, $\phi$ sectional curvature of a Sasakian manifold, semi-symmetric and Weyl semi-symmetric Sasakian manifolds, C-Bochner curvature tensor, $N(k)$-Contact Metric Manifolds: $k$-nullity distribution, $\eta$-Einstein $N(k)$-Contact Metric manifolds, Conformally flat $N(k)$-contact metric manifolds, some curvature properties, Almost para-contact structure, Torsion tensor fields, Examples of paracontact manifolds, $P$-Sasakian manifolds.

Submanifolds of Sasakian Manifolds: Invariant submanifolds of Sasakian manifolds, Anti-invariant submanifolds tangent to the structure vector field of Sasakian manifolds, Anti-invariant submanifolds normal to the structure vector field of Sasakian manifolds.

References:

1. R.S.Mishra; Structure on a Differentiable manifold and their Applications; Chandrama Prakashani, Allahabad, 1984.


Gravitational collapse of a homogeneous dust ball. Schwarzschild black hole. Simple idea of black hole physics.

References:

3. Introduction to cosmology - J.V. Narlikar.
5. Gravitation and Cosmology – S. Weinberg (J. Wiley and Sons.)
7. Introduction to Cosmology – M. Ross (J. Wiley and Sons).

N9: Advanced Quantum Mechanics

Fundamental ideas of quantum mechanics: Nature of the electromagnetic radiation; Wave-particle duality - double-slit experiment, quantum unification of the two aspects of light, matter waves; Wave functions and Schrodinger equation; Quantum description of particle - wave packet, uncertainty relation.

Mathematical formalism of quantum mechanics: Wave function space – bases, representation; State space – bases, representation; Observables – R and P observables; Postulates of quantum mechanics.

Physical interpretation of the postulates: Statistical interpretation – expectation values, Ehrenfest theorem, uncertainty principle; Physical implications of the Schrodinger equation - evolution of physical systems, superposition principle, conservation of probability, equation of continuity; Solution of the Schrodinger equation – time evolution operator, stationary state, time-independent Schrodinger equation; Equations of motion – Schrodinger picture, Heisenberg picture, interaction picture.

Theory of harmonic oscillator: Matrix formulation – creation and annihilation operators; Energy values; Matrix representation in n basis; Representation in the coordinate basis; Planck’s law; Oscillator in higher dimensions.
Symmetry and conservation laws: Symmetry transformations – basic concepts, examples; Translation in space; Translation of time; Rotation in space; Space inversion; Time reversal.

Angular momentum: Orbital angular momentum - eigen values and eigen functions of \( L_2 \) and \( L_z \); Angular momentum operators \( \hat{J} \) – commutation relations, eigen values and eigen functions; Representations of the angular momentum operators.

Spin: Idea of spin – Bosons, Fermions; Spin one-half – eigen functions, Pauli matrices; Total Hilbert space for spin-half particles; Addition of angular momenta; Clebsch-Gordan coefficients–computation, recursion relations, construction procedure; Identical particles - symmetrisation postulate, Pauli exclusion principle, normalization of states.

One-electron atom: Schrodinger equation; Energy levels, Eigen functions and bound states, Expectation values and virial theorem; Solution in parabolic coordinates; Special hydrogenic atom (brief description) – positronium, muonium, antihydrogen, Rydberg atoms.

Time-independent perturbation theory: Basic concepts; Derivation – up to the second order correction to the energy values and wave functions; Applications - anharmonic oscillator; normal helium atom, ground state of hydrogen and Stark effect. Variational method: Rayleigh-Ritz variational principle; Applications – one dimensional harmonic oscillator, hydrogen atom, helium atom.

Relativistic quantum mechanics: Klein-Gordon equation – plane wave solution, interpretation of K-G equation; Dirac equation – covariant form, charged particle in electromagnetic field, equation of continuity, plane wave solution; Dirac hole theory; Spin of the Dirac particle.

References:


N10: Boundary Layer Theory and Turbulence

Brief history and development of Fluid Mechanics, Fluid properties; Newtonian and non-Newtonian fluids, Different models of non-Newtonian fluids, Nanofluids and its different models, Integral laws for conservation of mass, momentum, angular momentum and energy; Constitutive laws, Differential forms of mass conservation equation, Navier-Stokes Equations; Differential form of Energy equation. Scaling and dimensional analysis, Dynamic Similarity, Laminar and Turbulent flows, Pipe flow, Open channel flow, Boundary layer theory, similarity solutions, high Re flows, creeping flows, steady and unsteady flows, heat transfer coefficients, molecular diffusion in fluids, mass transfer coefficients, Statistical interpretation of turbulence and its analysis, Spectral analysis and Kolmogorov theory.

Fundamentals of finite difference methods – explicit and implicit schemes; numerical stability and numerical solutions to non-linear ordinary and partial differential equations.
References:


N11: Chaos Theory and Fractals

Full Marks: 50 Credit: 4

Prerequisite: Ideas on dynamical systems both continuous and discrete systems, fixed points, periodic points, periodic cycles and their stabilities, bifurcations theory and some important maps.

Topological conjugacy, properties of conjugacy, semi-conjugacy relations.

Mathematical theory of chaos: Sensitive dependence on initial condition (SDIC), topological transitivity and mixing, definition of chaotic map. Examples of chaotic maps, ergodic map and ergodic theorem, dynamics of logistic map for \( r \geq 4 \), symbolic dynamics.

Quantifying chaos: Universal sequence, Feigenbaum number, renormalization group theory and super-stable cycle, Lyapunov exponent and invariant measure.

Sharkovskii’s theorem, Li and Yorke theorem, Poincare map, circle map and Smale Horseshoe map. Routes of chaos, universality in chaos.

Fractals: Self-similarity and scaling, self-similar fractals, constructions of self-similar fractals, dimensions of fractals, strange attractors, fractal basin boundary. Applications to fractals in chaotic dynamics and biological systems.

References:


N12: Non-Linear Partial Differential Equations

Full Marks: 50 Credit: 4

First-order non-linear PDE: First-order nonlinear equations and their applications, the generalized method of characteristics, complete integrals of certain special nonlinear equations, the Hamilton-Jacobi equation and its applications.
Second-order non-linear PDE: Non-linear model equations and variational principles, basic concepts and definitions, some nonlinear model equations, variational principles and the Euler-Lagrange equations, the variational principle for nonlinear Klein-Gordon equations.

Conservation laws: Conservation laws and shock waves, conservation laws, discontinuous solutions and shock waves, weak or generalized solutions.

References:


N13: Fuzzy Sets and Applications

Fuzzy sets: Basic definitions, level sets, convex fuzzy sets, basic operations on fuzzy sets, types of fuzzy sets, Cartesian products, algebraic products bounded sum and difference.

Extension principle and application: Zadeh extension principle, image and inverse image of fuzzy sets, fuzzy numbers, elements of fuzzy arithmetic.

Fuzzy relations: Fuzzy relations on fuzzy sets, union and intersection of fuzzy relation, composition of fuzzy relations, min-max composition and its properties, fuzzy equivalence relation.

Fuzzy decision: Fuzzy linear programming problems: Symmetric fuzzy linear programming problem, fuzzy linear programming with crisp objective function, fuzzy graph.

Fuzzy logic: An overview of classic logic, its connectives, tautologies, contradiction fuzzy logic, fuzzy quantities, logical connectives for fuzzy logic, applications to control theory.

References:


N14: Mathematical Logic

Programming Prolog - facts, simple queries, complex queries, rules, arithmetic operators, recursion, unification, lists, cut. Propositional logic - syntax, semantics, laws of deduction, normal forms, resolution, theorems proving, validity, soundness and completeness. First order logic - conversion of common sense sentences into the language of first order logic, universal and existential quantifiers, syntax, terms of predicate, model theoretic semantics, Harbrand universe, normal form, unification, proof theory, mechanical theorems proving, incompleteness.
References:

N15: Multivariate Analysis

Multiple linear regression, problems of multicollinearity, heteroscedasticity and their remedies, autocorrelation and its remedial measures, polynomial regression and the method of orthogonal polynomials, nonlinear regression - some specific models and solutions. Testing general linear hypothesis - testing equality of means of several normal distributions with common covariance matrix, MANOVA, testing independence of a set of variates, testing hypotheses of equality of covariance matrices. Confounding in factorial experiments, fractional replications, split-plot designs. BIBD and PBIBD. Factor analysis.

References:

N16: Numerical Optimization

Basics of numerical optimization algorithms and their convergence properties.

Derivative free algorithms and their convergence analysis: (I) Single Dimension-Golden section and Fibonacci search methods (II) Multidimension- Coordinate search method, Method of Hookes and Jeeves, Method of Rosenberg.

Derivative based algorithms and their convergence analysis: Steepest descent method, Newton method, Conjugate gradient methods, Quasi Newton methods, Trust region methods, Penalty, Barrier, and Augmented Lagrangian methods, Sequential Quadratic Programming Technique.

References:

N17: Lie Groups and Lie Algebras

Transformation groups, orthogonal groups in 3- space, Euclidean group E(2), symmetry and discrete groups of E(3), group representation, reducible and irreducible representations, group characters, exponential of a
matrix, local Lie groups, classical groups, Lie algebras, solvable and nilpotent Lie algebras, semi simple Lie algebras, classical Lie algebras, exponential map of a Lie algebra.

References:


N18: Queuing Theory

Full Marks: 50 Credit: 4

Probability and random variable, discrete and continuous univariate and multivariate distributions, moments, law of large numbers and central limit theorem (without proof), Poisson process, birth and death process, infinite and finite queueing models M/M/1, M/M/C, M/G/1, M/M/1/N, M/Ek/1, Ek/M/1, M/G/1/N, GI/M/1, priority queueing models, network of queues, finite processor sharing models, central server model of multiprogramming, performance evaluation of systems using queueing models. Concepts of bottleneck and system saturation point. Introduction to discrete time queues and its applications.

References:


N19: Data Structure and Algorithm

Full Marks: 50 Credit: 4


References: